Computer-based Design of Space-Time Codes

François Chan
Dept. of Elec. & Comp. Eng.
Royal Military College of Canada
Email: chan-f@rmc.ca

Queen’s Colloquium in Computational Science and Engineering

Feb. 26, 2008

Outline

1. Introduction
2. MIMO (Multiple Input Multiple Output) systems
3. Design of space-time trellis codes
4. Super-orthogonal space-time trellis codes
5. Conclusion

1. Introduction

- **Modulation**: conversion or mapping of a discrete-time sequence of symbols into continuous-time signals suitable for transmission by physical channel.
- Involves varying some characteristics of a carrier (sinusoidal wave) in accordance with a message.

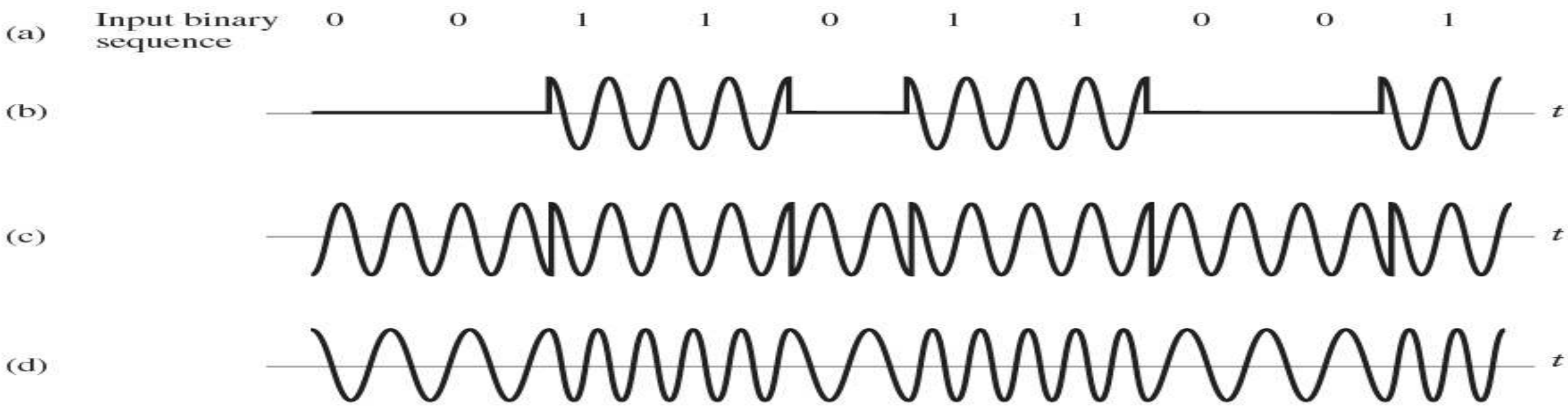


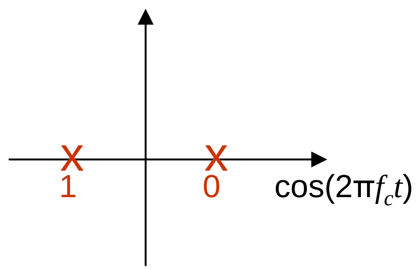
FIGURE 7.1 The three basic forms of signaling binary information. (*a*) Binary data stream. (*b*) Amplitude-shift keying. (*c*) Phase-shift keying. (*d*) Frequency-shift keying with continuous phase.

Modulation (cont'd)

- BPSK: Binary Phase-Shift Keying

$$s_i(t) = \begin{cases} \cos(2\pi f_c t), & \text{for symbol '0'} \\ \cos(2\pi f_c t + \pi) = -\cos(2\pi f_c t), & \text{for symbol '1'} \end{cases}$$

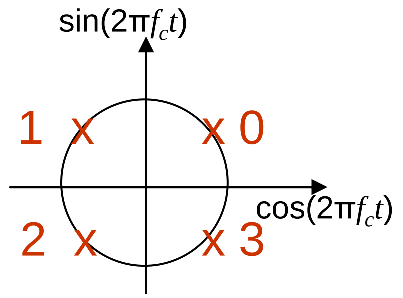
  where $0 \le t \le T$



- QPSK: Quadrature Phase-Shift Keying

$$s_i(t) = \cos\left[2\pi f_c t + (2i+1)\frac{\pi}{4}\right]$$

  where $0 \le t \le T$ and $i = 0, 1, 2, 3$.



- M-ary PSK
- M-ary QAM (Quadrature Amplitude Modulation)

Rayleigh fading channel

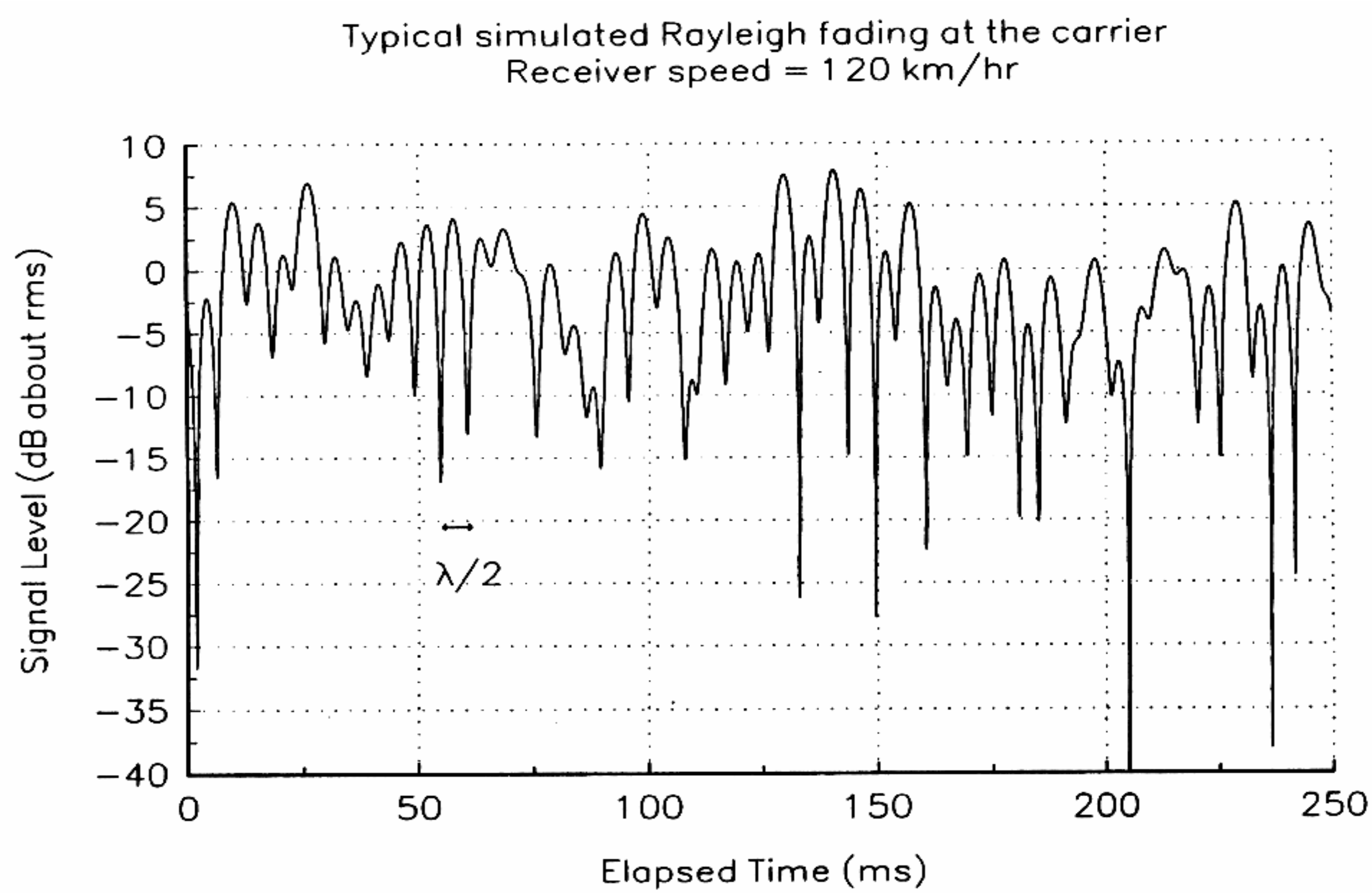
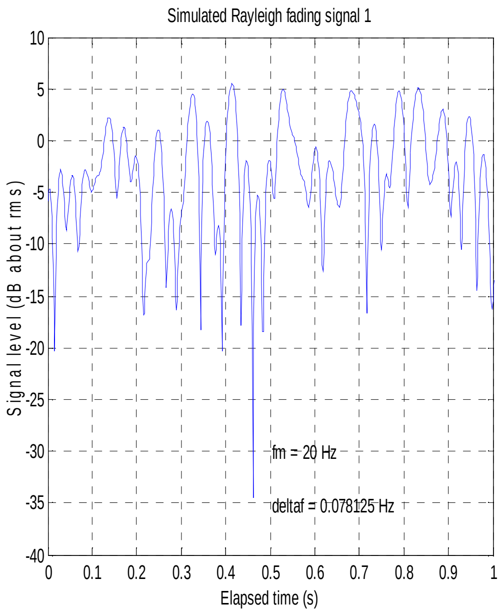


Figure 5.15 A typical Rayleigh fading envelope at 900 MHz [from [Fun93] © IEEE].

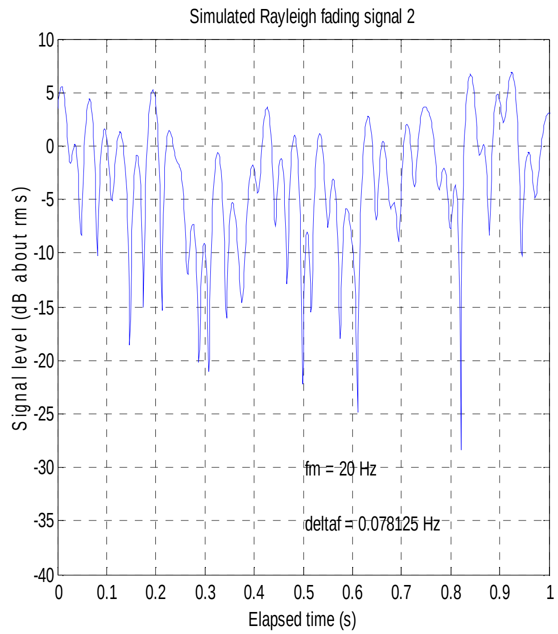
Diversity

- *Diversity* is a technique used to compensate for fading channel impairments: it can reduce the depth and duration of the fades.
- Diversity techniques include:
  - Antenna polarization diversity and angular diversity.
  - Frequency diversity: replicas of the signal transmitted at different frequencies, OFDM (Orthogonal Frequency Division Multiplexing).
  - Time diversity: error-correcting coding, RAKE receiver for CDMA spread spectrum systems (correlation receiver for each of the multipath signals), interleaving.
  - Space diversity: multiple antennas are used at the transmitter and receiver.

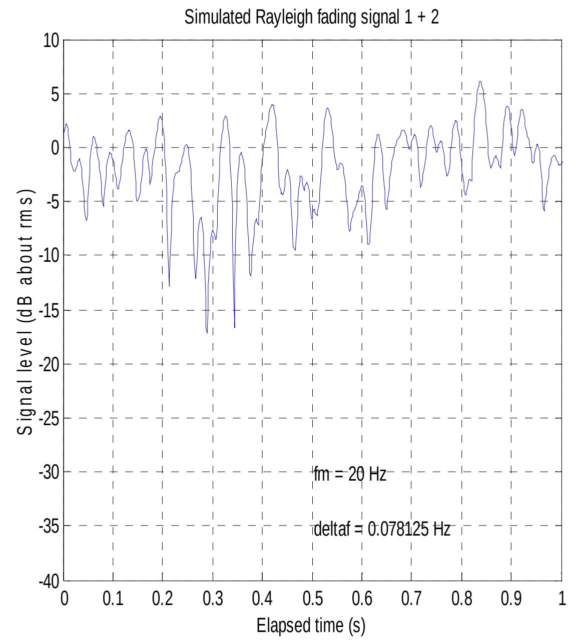
Diversity means two paths are better than one



+



=



2. MIMO (Multiple Input Multiple Output) systems

Space Diversity

- Space or antenna diversity at the receiver has been known and used for many years.
- Space diversity at the transmitter is relatively new: Foschini (1996), Tarokh, Seshadri & Calderbank (1998).
- It can be shown that the data rate of a wireless communications system can be significantly increased by deploying multiple transmit and receive antennas.
- If the numbers of transmit and receive antennas are the same, the capacity increases at least linearly with the number of antennas.

Transmit diversity

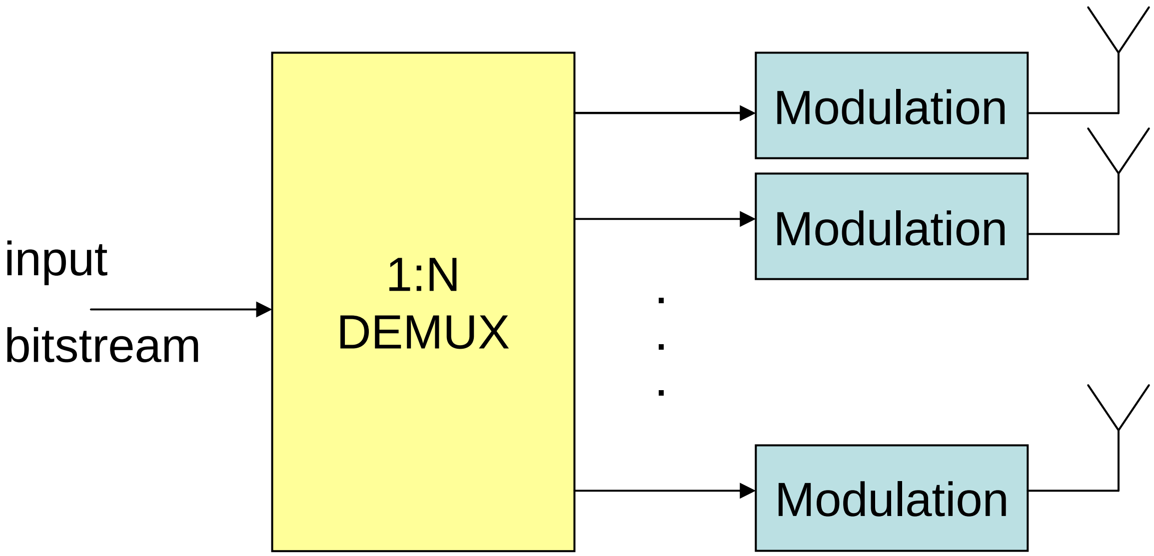
- Beamforming

  Requires channel state information (CSI) at transmitter
- Spatial multiplexing

  Powerful technique for increasing data rate at high signal-to-noise ratios.
- Diversity coding
  - Space-time block codes
  - Space-time trellis codes

Spatial multiplexing

- Spatial multiplexing is used to increase the throughput of communication systems with multiple antennas.
- V-BLAST (Vertical – Bell Labs Layered Space-Time) is a practical approach to achieve spatial multiplexing.
- Detection includes three steps: ordering, interference cancellation, interference nulling.



A simple example of spatial multiplexing (V-BLAST)

Orthogonal space-time block codes

- Alamouti code (N = 2 transmit antennas)

Antenna 1   Antenna 2

$$C = \begin{pmatrix} s_1 & s_2 \\ -s_2^* & s_1^* \end{pmatrix} \quad \begin{matrix} \text{time 1} \\ \text{time 2} \end{matrix}$$

- Significant performance improvement compared to the system with one transmit antenna.
- Simple decoding

$$\tilde{s}_1 = \sum_{m=1}^{M} \left[r_{1,m}\alpha_{1,m}^* + r_{2,m}\alpha_{2,m} \right]$$

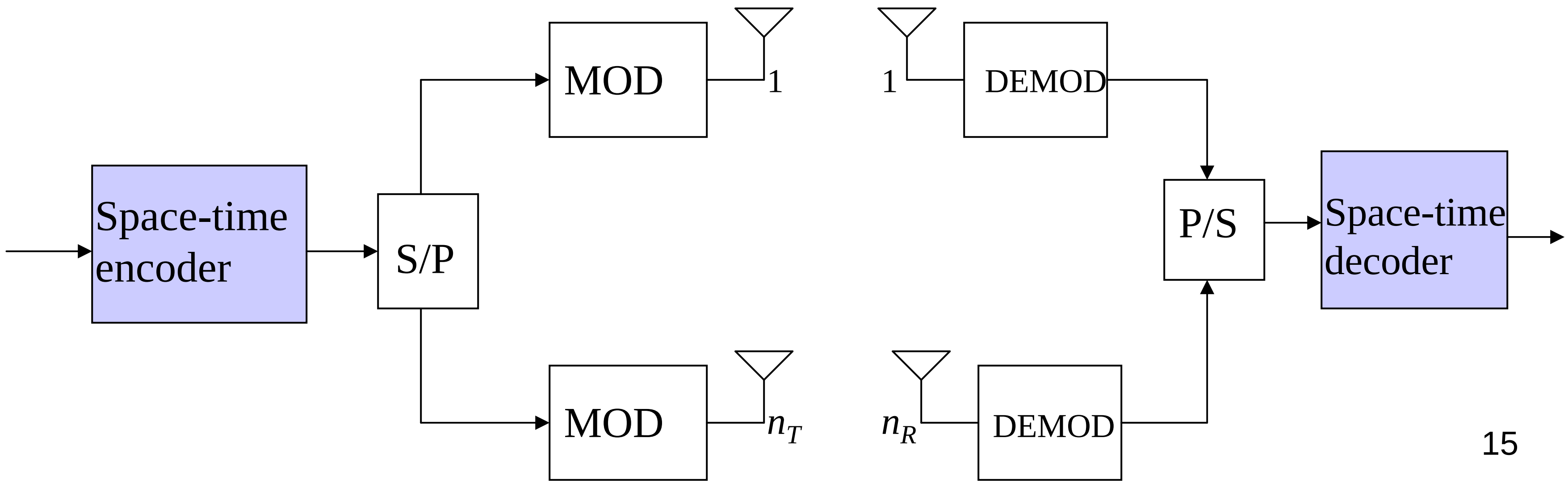
$$\tilde{s}_2 = \sum_{m=1}^{M} \left[r_{1,m}\alpha_{2,m}^* - r_{2,m}\alpha_{1,m} \right]$$

α: path gain, r_1: signal received at time 1

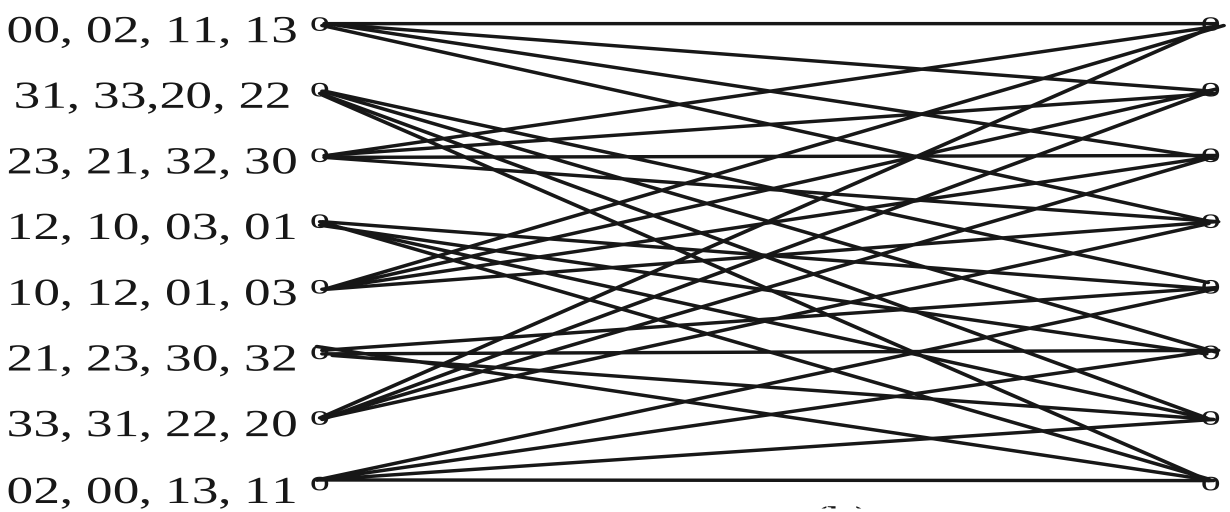
- Space-time block codes provide diversity gain and simple decoding.
- For N=2 (Alamouti code), full rate or R=1, i.e., 2 symbols are transmitted in 2 time slots, same as system with N=1 antenna.
- No full rate complex orthogonal space-time block code for N>2 transmit antennas $\rightarrow R < 1$
- No coding gain.

Space-time trellis codes

- A space-time trellis code is a Trellis-Coded Modulation (TCM) scheme for multiple transmit antennas.
- Space-time coding can significantly improve the performance and the data rate of a wireless communications system.



- Example: 8-state space-time trellis codes for 2 transmit antennas and QPSK modulation



- Decoding is done using the Viterbi algorithm.

3. Design of space-time trellis codes

Matrix representation

- For *M*-PSK, number of information bits transmitted during each use of the channel $m = \log_2 M$
- information sequence $\mathbf{u} = (u_1, \ldots, u_n)$, where $n = l \log_2 M$, l: number of symbols out of each antenna and u_i=0 or 1.
- s: number of memory elements in the encoder (number of states = 2^s)
- sequence of coded PSK symbols

$$\mathbf{c} = (\mathbf{c}_1, \cdots, \mathbf{c}_l) = (c_1^1 c_1^2 \cdots c_1^{n_T} \cdots c_l^1 c_l^2 \cdots c_l^{n_T})$$

- coded symbol at instant t: $\mathbf{c}_t = \mathbf{u}_t \mathbf{G} \pmod{M}$

  where $\mathbf{u}_t$: $(m+s)$ input bits influencing $\mathbf{c}_t$

$\mathbf{u}_t = (u_{mt+(m-1)}\, u_{mt+(m-2)} \ldots u_{mt-s})$

G: generator matrix with n_T columns and $m+s$ rows (each element of **G** is between 0 and M-1)

Systematic code search

- $\Rightarrow$ examine all possible $M^{[n_T(m+s)]}$ matrices **G**.
- Example:

  For QPSK with n_T=2:

  - 4 states: $4^{[2(2+2)]} = 4^8 = 65536$ matrices **G**
  - 8 states: $4^{[2(3+2)]} = 4^{10} = 1{,}048{,}576$ matrices **G**
- For codes with up to 32 states: exhaustive search
- For codes with 64 states and more: random search.

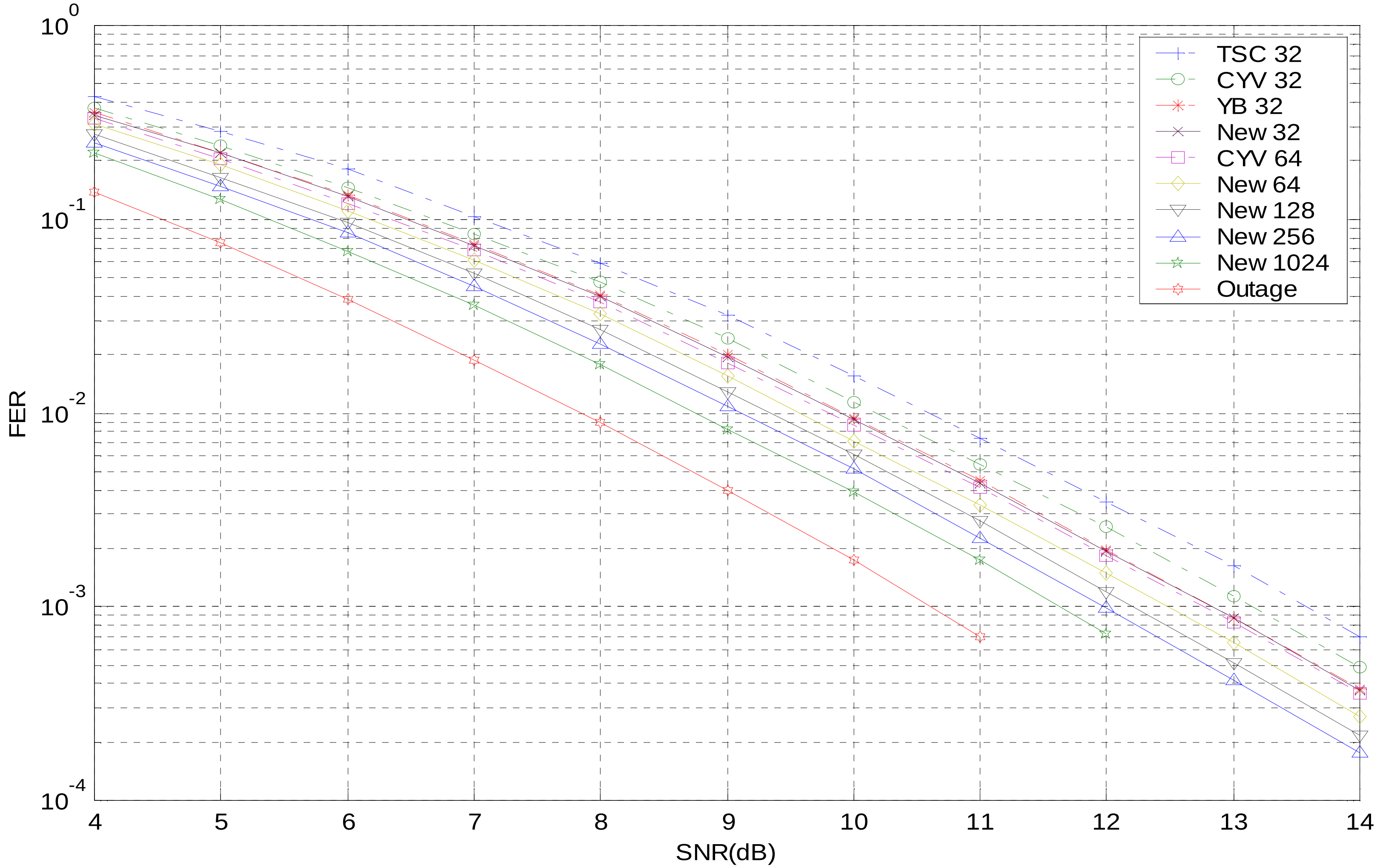
Systematic code search (cont'd)

- Iterative process
- 1st iteration:
  - Generate matrix **G**
  - Compute determinant and trace of all difference matrices.
  - If minimum determinant and trace satisfy threshold, simulate code using a modest number of frames.
- 2nd iteration: simulate best codes (either an arbitrary number or those satisfying a given probability of error) using a larger number of frames.
- 3rd iteration: simulate best codes with an even larger number of frames.
- Parallel processing.

- Main cluster: seven Sun Fire 25000 servers each of which has 72 dual-core (CPU) UltraSPARC-IV+ processors with 576 GB of RAM; also connected using Gigabit Ethernet.
- Additional three Sun Fire 15000 servers configured with 72 UltraSPARC-III processors and 288 GB of memory.
- A total of 160 TB of Sun StorEdge 3510 disk.

New QPSK space-time trellis codes for 2 transmit antennas

David Bernier & Francois Chan



Punctured Space-Time Convolutional Codes for Adaptive Modulation Schemes

David Bernier & Francois Chan

- Example: Conventional 4-state QPSK code for 2 antennas

  Let $\mathbf{G}^T = \begin{pmatrix} 2 & 0 & 1 & 2 \\ 2 & 2 & 2 & 1 \end{pmatrix}$ (Yan & Blum, 2002)

  Let the information sequence be $\mathbf{u_t}$ = (1001)

  Then, $\mathbf{c}_t = \mathbf{u_t}\,\mathbf{G}$ (mod 4) = (0 3)

- Example: mod 2 code

  Let $\mathbf{u_t}$ = (1001) and $\mathbf{G} = \begin{pmatrix} 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 \end{pmatrix}$

  Then, $\mathbf{c}_t = \mathbf{u_t}\,\mathbf{G}$ (mod 2) = (0001).

  $\mathbf{c}_t$ is mapped into the two QPSK symbols 0 and 1.

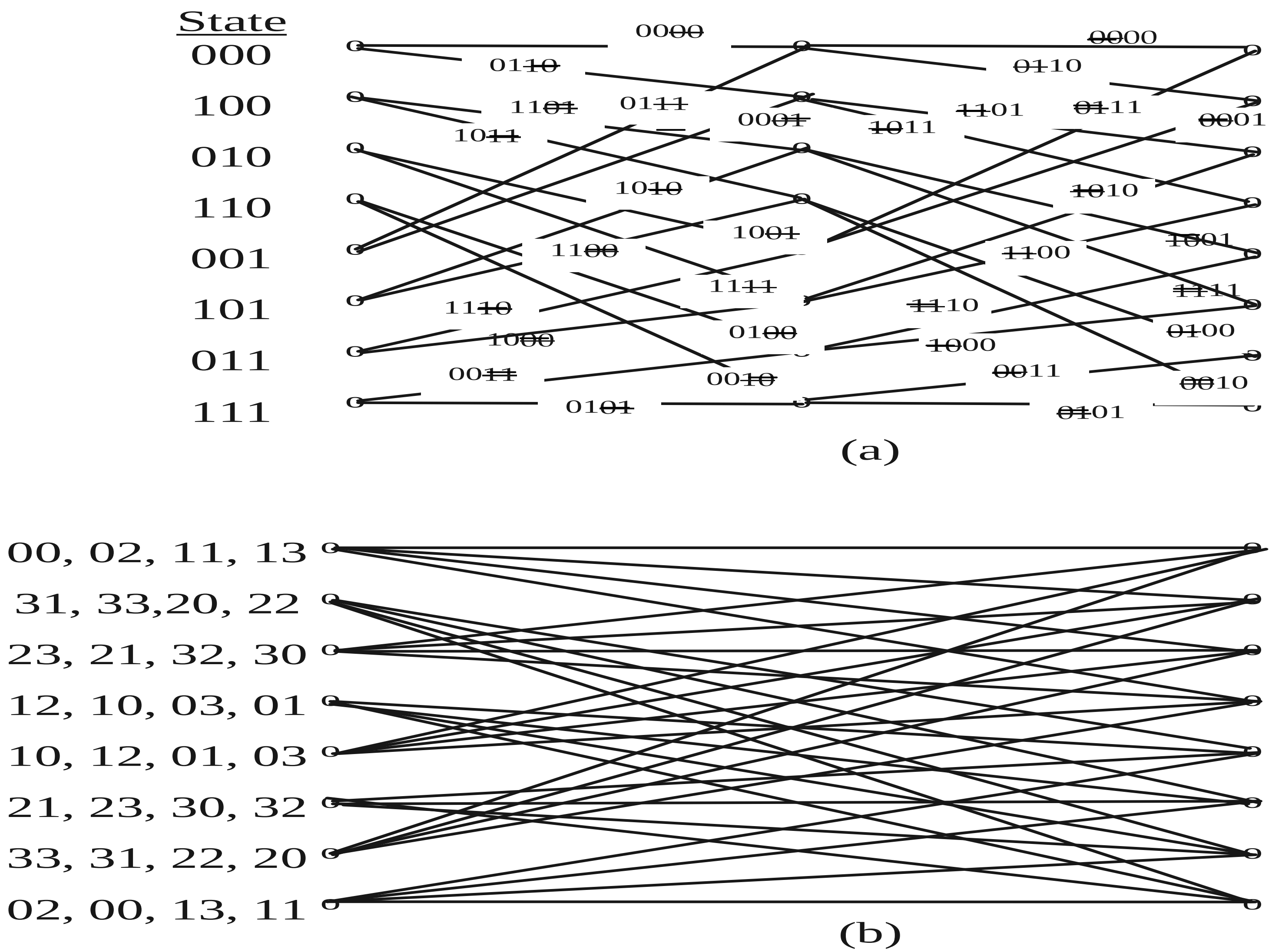


Fig. 4 (a) Trellis of the original rate-1/4, 8-state convolutional code; punctured bits are struck.

(b) Trellis of the rate-2/4 code after puncturing and QPSK mapping – 2 b/s/Hz.

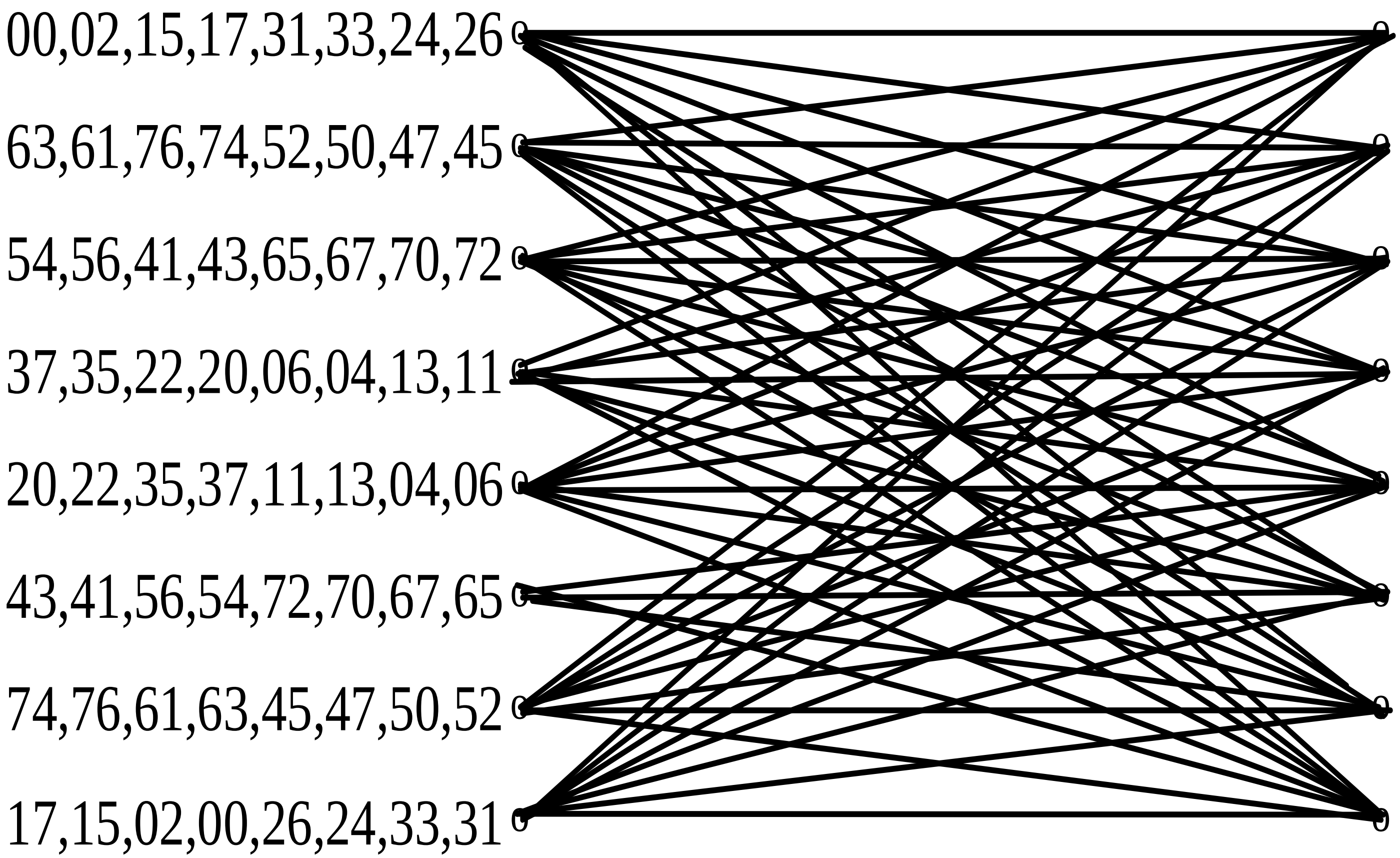
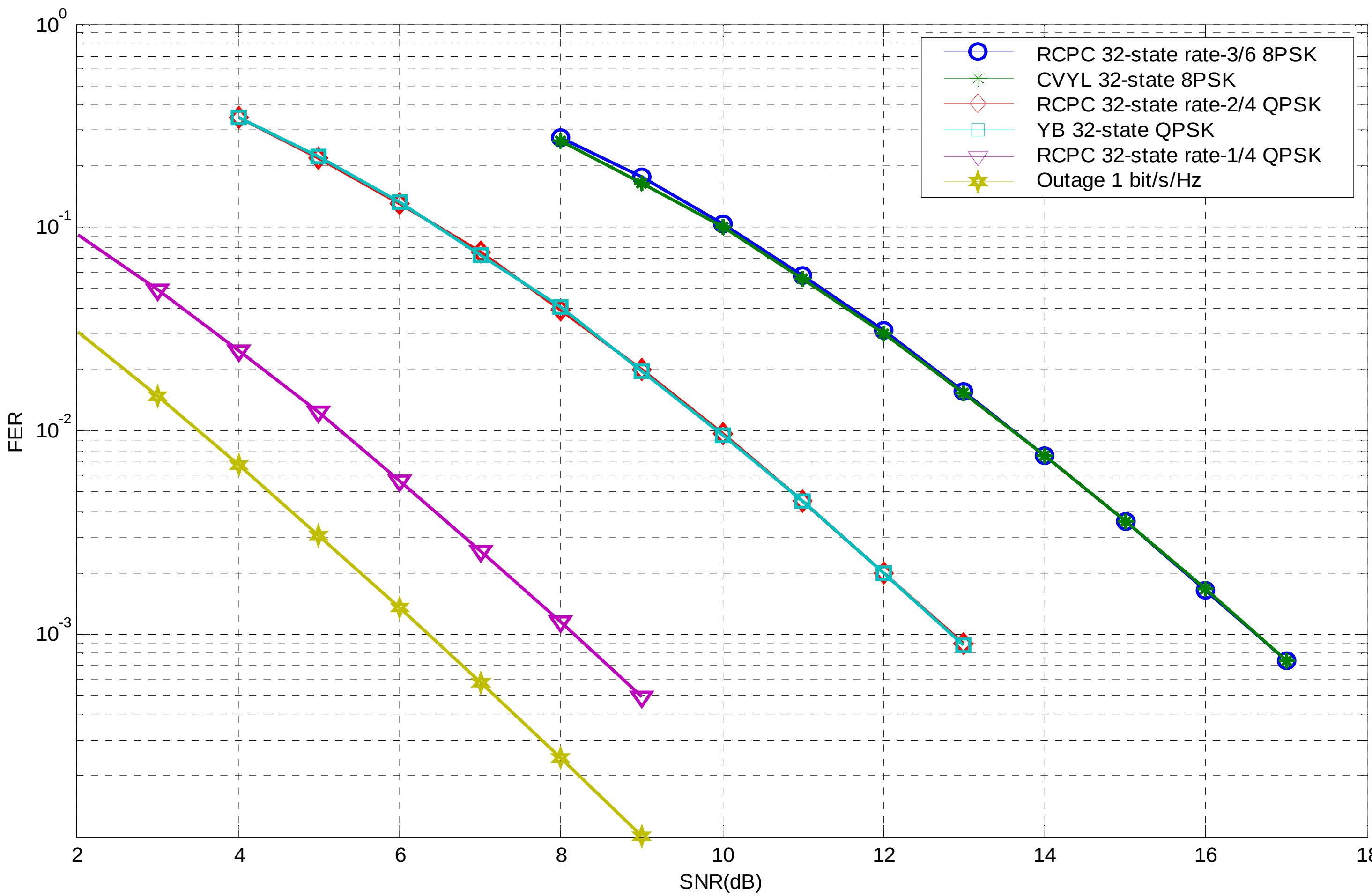


Fig. 5. Trellis of a rate-3/6 code after puncturing the original rate-1/4 code with P = [1 1 0 0   0 1 1 0   0 0 1 1] and 8PSK mapping – 3 b/s/Hz.

Comparison of the error performance of 32-state punctured space-time trellis codes, CVYL codes [10] and YB codes [4] - 2 transmit and 2 receive antennas



4. Super-Orthogonal Space-Time Trellis Codes (SOSTTC)

- Concatenate inner space-time block code with outer trellis code
- Expand set of block codes without expanding signal constellation
- Maintain block code orthogonality

$$C(x_1, x_2, \theta) = \begin{pmatrix} x_1 e^{j\theta} & x_2 \\ -x_2^* e^{j\theta} & x_1^* \end{pmatrix} \qquad \begin{aligned} &\theta = 2\pi l'/M \\ &l' = 0, 1, \cdots, M-1 \end{aligned}$$

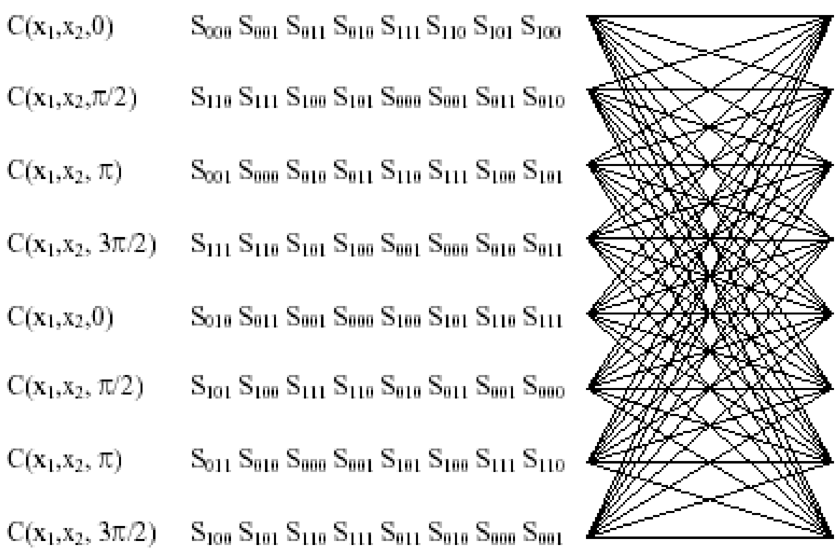
Computer Design of Super-Orthogonal Space-Time Trellis Codes

Michael Bale, Brady Laska, Dustin Dunwell, Francois Chan and Hamid Jafarkhani

- Objective:
  - Find optimal codes
  - Facilitate design of codes with large number of states
- Solution:
  - Represent codes by a simple generator matrix
  - Exhaustive computer search for optimal codes

Complexity Comparison

Traditional Representation:



Generator Matrix Alternative:

$$\mathbf{G} = \begin{pmatrix} a_1 & a_2 \\ a_3 & a_4 \\ a_5 & a_6 \\ a_7 & a_8 \\ a_9 & a_{10} \\ a_{11} & a_{12} \\ a_{13} & a_{14} \end{pmatrix}$$

How it Works

- Information Sequence to be transmitted:
  - **u** = (u_1, …u_n) can be represented as vector $\mathbf{u}_l$ where l is the trellis level
  - $\mathbf{u}_l$ then considered to be a shift register

$$\mathbf{u}_l = \begin{bmatrix} \overbrace{u_7 \quad u_6 \quad u_5}^{NextState} & \overbrace{u_4}^{ParallelBranch} & \overbrace{u_3 \quad u_2 \quad u_1}^{CurrentState} \end{bmatrix}$$

  - Multiply $\mathbf{u}_l$ by generator matrix to yield x_1 and x_2
  - Map these symbols to transmission matrix

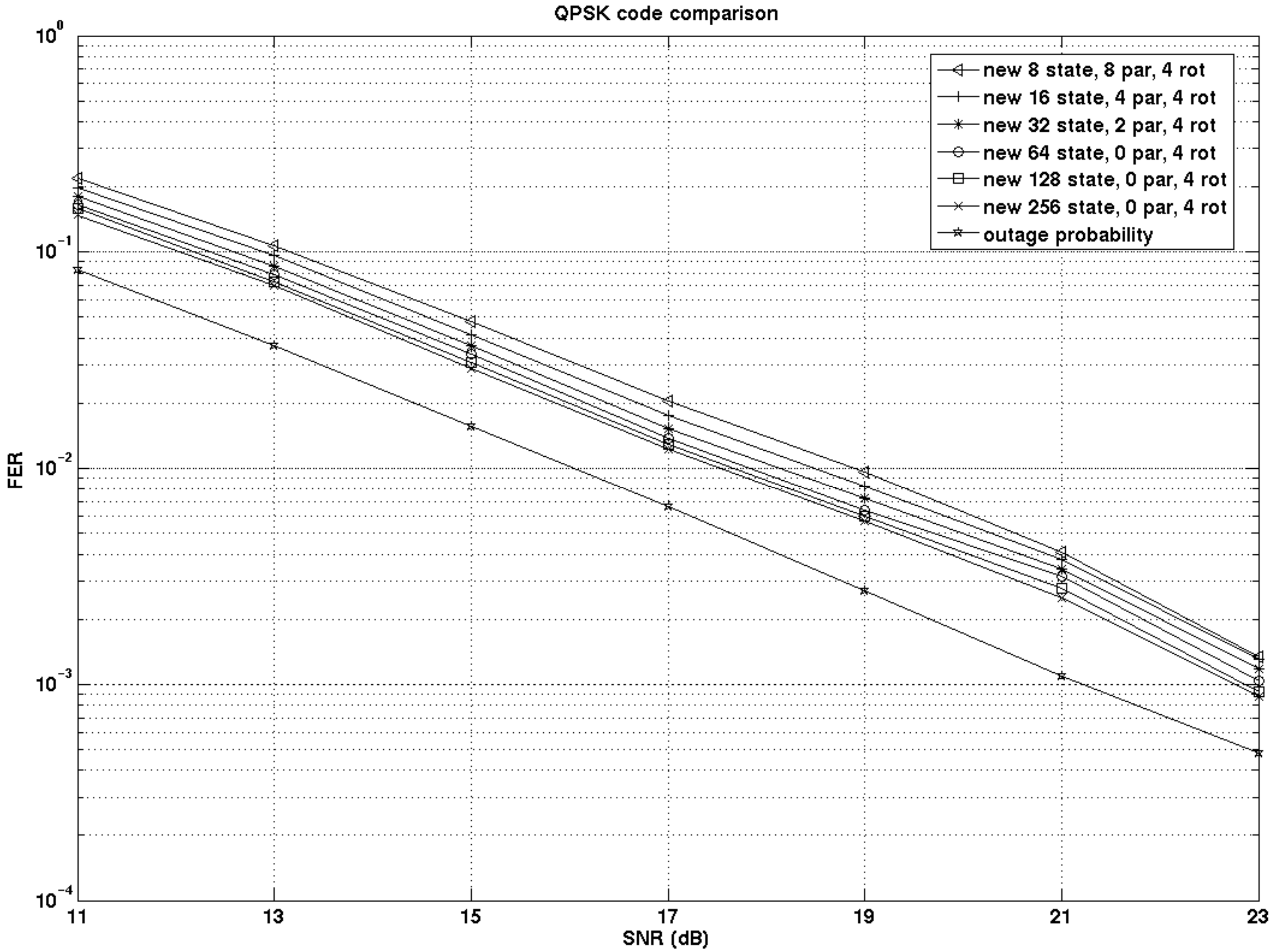
Example

- 8 State Code with 2 Parallel Branches

$$\mathbf{u}_1 = \begin{bmatrix} \overbrace{001}^{NextState} & \overbrace{1}^{ParallelBranch} & \overbrace{010}^{CurrentState} \end{bmatrix} \text{x} \begin{bmatrix} 2 & 1 \\ 3 & 1 \\ 0 & 2 \\ 2 & 2 \\ 3 & 3 \\ 0 & 2 \\ 2 & 3 \end{bmatrix} = (2\,,\,2)$$

- (2 , 2) gets mapped to the transition from state 1 to state 2 on parallel branch 2

Performance of the new best SOSTTC QPSK codes

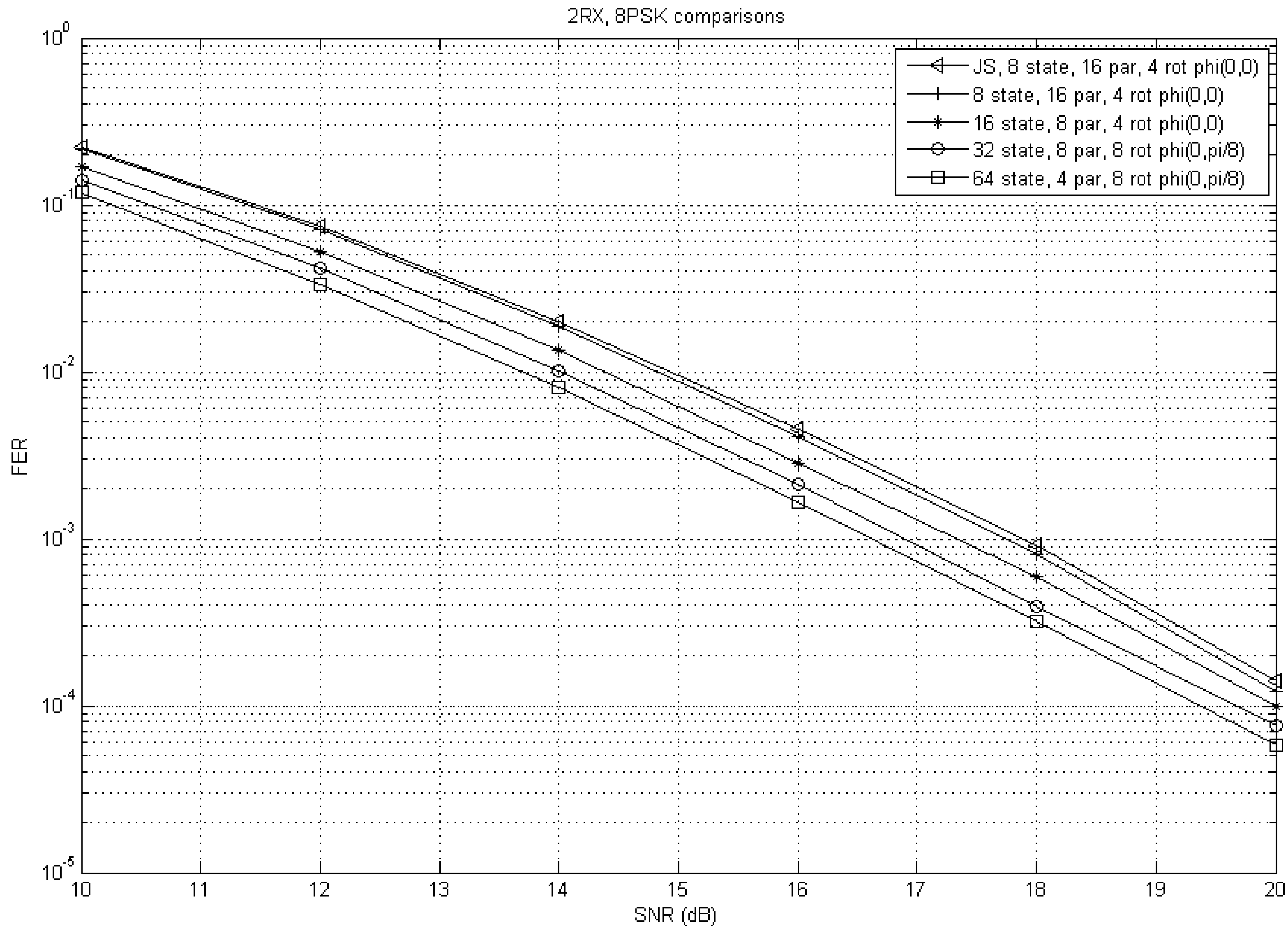


Design Rules for Extended Super-Orthogonal Space-Time Trellis Codes

Elizabeth Rivera Hartling, Francois Chan and Hamid Jafarkhani

- Design of SOSSTTCs becomes prohibitively complex as the number of states or the constellation size increases
- Identification of duplicate or equivalent codes
- Rules for the design of the generator matrix
- Complexity reduction
  - Example: 8PSK SOSTTC with 8 states, 8 parallel branches
  - Number of codes: 8^{18}
  - Reduced to 49,152.

Performance of 8PSK SOSTTC with 8 to 64 states – 2 receive antennas, 3 b/s/Hz



5. Conclusion

- MIMO is one of the hottest and most promising areas of research in wireless communications.
- It will be used in most future wireless systems: 802.11n, 3G, etc.

Future research

- Application: MIMO-OFDM
- Multi-User MIMO
- Cooperative MIMO